CURRICULUM GRID

# Year 11 Maths Methods for the Australian Curriculum

## Unit 1

### Unit description

This unit begins with a review of the basic algebraic concepts and techniques required for a successful introduction to the study of calculus. The basic trigonometric functions are then introduced. Simple relationships between variable quantities are reviewed, and these are used to introduce the key concepts of a function and its graph. The study of inferential statistics begins in this unit with a review of the fundamentals of probability and the introduction of the concepts of conditional probability and independence. Access to technology to support the computational aspects of these topics is assumed.

### Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques in algebra, functions, graphs, trigonometric functions and probability
- solve problems using algebra, functions, graphs, trigonometric functions and probability
- apply reasoning skills in the context of algebra, functions, graphs, trigonometric functions and probability
- interpret and evaluate mathematical information and ascertain the reasonableness of solutions to problems
- communicate their arguments and strategies when solving problems.

| Unit 1 Content descriptions | | |
|---|---|---|
| **Topic 1: Functions and graphs** | | |
| **Subtopics** | **Curriculum statements** | **Textbook sections** |
| **Lines and linear relationships** | • determine the coordinates of the midpoint of two points (ACMMM001) AC | 4.03 |
| | • examine examples of direct proportion and linearly related variables (ACMMM002) AC | 1.03, 1.04, 1.09 |
| | • recognise features of the graph of $y = mx + c$, including its linear nature, its intercepts and its slope or gradient (ACMMM003) AC | 1.04, 4.03, 4.04 |
| | • find the equation of a straight line given sufficient information; parallel and perpendicular lines (ACMMM004) AC | 4.04 |
| | • solve linear equations (ACMMM005) AC | 1.03, 1.05, 1.06 |
| **Review of quadratic relationships** | • examine examples of quadratically related variables (ACMMM006) AC | 1.01, 1.02, 1.07, 4.05 |
| | • recognise features of the graphs of $y = x^2$, $y = a(x - b)^2$ and $y = a(x - b)(x - c)$, including their parabolic nature, turning points, axes of symmetry and intercepts (ACMMM007) AC | 4.05, 4.07 |
| | • solve quadratic equations using the quadratic formula and by completing the square (ACMMM008) AC | 1.02, 1.07, 1.08 |
| | • find the equation of a quadratic given sufficient information (ACMMM009) AC | 1.06 |
| | • find turning points and zeros of quadratics and understand the role of the discriminant (ACMMM010) AC | 1.08, 4.05, 4.06 |
| | • recognise features of the graph of the general quadratic $y = ax^2 + bx + c$ (ACMMM011) AC | 4.05, 4.06 |
| **Inverse proportion** | • examine examples of inverse proportion (ACMMM012) AC | 1.09, 4.07 |
| | • recognise features of the graphs of $y = \frac{1}{x}$ and $y = \frac{a}{x - b}$, including their hyperbolic shapes, and their asymptotes (ACMMM013) AC | 4.07 |
| **Powers and polynomials** | • recognise features of the graphs of $y = x^n$ for $n \in N$, $n = -1$ and $n = \frac{1}{2}$, including shape, and behaviour as $x \to \infty$ and $x \to -\infty$ (ACMMM014) AC | 1.04, 4.03–4.07 |
| | • identify the coefficients and the degree of a polynomial (ACMMM015) AC | 1.10–1.12 |
| | • expand quadratic and cubic polynomials from factors (ACMMM016) AC | 1.01, 1.10–1.12 |
| | • recognise features of the graphs of $y = x^3$, $y = a(x - b)^3 + c$ and $y = k(x - a)(x - b)(x - c)$, including shape, intercepts and behaviour as $x \to \infty$ and $x \to -\infty$ (ACMMM017) AC | 4.07 |
| | • factorise cubic polynomials in cases where a linear factor is easily obtained (ACMMM018) AC | 1.11 |
| | • solve cubic equations using technology, and algebraically in cases where a linear factor is easily obtained (ACMMM019) AC | 1.12 |
| **Graphs of relations** | • recognise features of the graphs of $x^2 + y^2 = r^2$ and of $(x - a)^2 + (y - b)^2 = r^2$, including their circular shapes, their centres and their radii (ACMMM020) AC | 4.08 |
| | • recognise features of the graph of $y^2 = x$ including its parabolic shape and its axis of symmetry (ACMMM021) AC | 4.08 |
| **Functions** | • understand the concept of a function as a mapping between sets, and as a rule or a formula that defines one variable quantity in terms of another (ACMMM022) AC | 4.01 |

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| Unit 1 Content descriptions | | |
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| **Topic 1: Functions and graphs** | | |
| **Subtopics** | **Curriculum statements** | **Textbook sections** |
| **Functions** | • use function notation, domain and range, independent and dependent variables (ACMMM023) AC | 1.04, 4.01 |
| | • understand the concept of the graph of a function (ACMMM024) AC | 1.04, 1.05, 4.01, 4.02 |
| | • examine translations and the graphs of $y = f(x) + a$ and $y = f(x + b)$ (ACMMM025) AC | 4.02 |
| | • examine dilations and the graphs of of $y = cf(x)$ and $y = f(kx)$ (ACMMM026) AC | 4.02 |
| | • recognise the distinction between functions and relations, and the vertical line test (ACMMM027) AC | 4.01 |

| Unit 1 Content descriptions | | |
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| **Topic 2: Trigonometric functions** | | |
| **Subtopics** | **Content statements** | **Textbook sections** |
| **Cosine and sine rules** | • review sine, cosine and tangent as ratios of side lengths in right-angled triangles (ACMMM028) AC | 3.01 |
| | • understand the unit circle definition of cos (θ), sin (θ) and tan (θ) and periodicity using degrees (ACMMM029) AC | 3.02 |
| | • examine the relationship between the angle of inclination of a line and the gradient of that line (ACMMM030) AC | 3.06 |
| | • establish and use the sine and cosine rules and the formula $Area = \frac{1}{2} bc \sin(A)$ for the area of a triangle (ACMMM031) AC | 3.03, 3.05, 3.06 |
| **Circular measure and radian measure** | • define and use radian measure and understand its relationship with degree measure (ACMMM032) AC | 3.07, 3.08 |
| | • calculate lengths of arcs and areas of sectors in circles (ACMMM033) AC | 3.08 |
| **Trigonometric functions** | • understand the unit circle definition of cos (θ), sin (θ) and tan (θ) and periodicity using radians (ACMMM034) AC | 6.01 |
| | • recognise the exact values of sin (θ), cos (θ) and tan (θ) at integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$ (ACMMM035) AC | 3.02, 3.07, 6.05 |
| | • recognise the graphs of $y = \sin(x)$, $y = \cos(x)$ and $y = \tan(x)$ on extended domains (ACMMM036) AC | 6.01–6.04 |
| | • examine amplitude changes and the graphs of $y = a \sin(x)$ and $y = a \cos(x)$ (ACMMM037) AC | 6.02–6.04 |
| | • examine period changes and the graphs of $y = \sin(bx)$, $y = \cos(bx)$ and $y = \tan(bx)$ (ACMMM038) AC | 6.03 |
| | • examine phase changes and the graphs of $y = \sin(x + c)$, $y = \cos(x + c)$ and $y = \tan(x + c)$ (ACMMM039) AC | 6.04 |
| | • the relationships $\sin(x + \frac{\pi}{2}) = \cos(x)$ and $\cos(x - \frac{\pi}{2}) = \sin(x)$ (ACMMM040) AC | 6.04 |
| | • prove and apply the angle sum and difference identities (ACMMM041) AC | 6.05 |
| | • identify contexts suitable for modelling by trigonometric functions and use them to solve practical problems (ACMMM042) AC | 6.07 |
| | • solve equations involving trigonometric functions using technology, and algebraically in simple cases (ACMMM043) AC | 6.06 |

| Unit 1 Content descriptions | | |
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| **Topic 3: Counting and probability** | | |
| **Subtopics** | **Content statements** | **Textbook sections** |
| **Combinations** | • understand the notion of a combination as an ordered of $r$ objects taken from a set of $n$ distinct objects (ACMMM044) AC | 2.03 |
| | • use the notation $\binom{n}{r}$ and the formula $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ for the number of combinations of $r$ objects taken from a set of $n$ distinct objects (ACMMM045) AC | 2.03 |
| | • expand $(x + y)^n$ for small positive integers $n$ (ACMMM046) AC | 2.02 |
| | • recognise the numbers $\binom{n}{r}$ as binomial coefficients, (as coefficients in the expansion of $(x + y)^n$) (ACMMM047) AC | 2.02 |
| | • use Pascal's triangle and its properties (ACMMM048) AC | 2.02 |
| **Language of events and sets** | • review the concepts and language of outcomes, sample spaces and events as sets of outcomes (ACMMM049) AC | 2.04 |
| | • use set language and notation for events, including $\overline{A}$ (or $A'$) for the complement of an event $A$, $A \cap B$ for the intersection of events $A$ and $B$, and $A \cup B$ for the union, and recognise mutually exclusive events (ACMMM050) AC | 2.01, 2.04 |
| | • use everyday occurrences to illustrate set descriptions and representations of events, and set operations (ACMMM051) AC | 2.01, 2.04 |
| **Review of the fundamentals of probability** | • review probability as a measure of 'the likelihood of occurrence' of an event (ACMMM052) AC | 2.05 |
| | • review the probability scale: $0 \leq P(A) \leq 1$ for each event $A$, with $P(A) = 0$ if $A$ is an impossibility and $P(A) = 1$ if $A$ is a certainty (ACMMM053) AC | 2.05–2.07 |
| | • review the rules: $P(A) = 1 - P(A)$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (ACMMM054) AC | 2.06–2.08 |
| | • use relative frequencies obtained from data as point estimates of probabilities (ACMMM055) AC | 2.08 |
| **Conditional probability and independence** | • understand the notion of a conditional probability and recognise and use language that indicates conditionality (ACMMM056) AC | 5.01–5.04, 5.06 |
| | • use the notation $P(A\|B)$ and the formula $P(A \cap B) = P(A\|B)P(B)$ (ACMMM057) AC | 5.04, 5.06, 5.07 |
| | • understand the notion of independence of an event $A$ from an event $B$, as defined by $P(A\|B) = P(A)$ (ACMMM058) AC | 5.05–5.07 |
| | • establish and use the formula $P(A \cap B) = P(A)P(B)$ for independent events $A$ and $B$, and recognise the symmetry of independence (ACMMM059) AC | 5.05–5.07 |
| | • use relative frequencies obtained from data as point estimates of conditional probabilities and as indications of possible independence of events (ACMMM060) AC | 5.07 |

# UNIT 2

## Unit description

The algebra section of this unit focuses on exponentials and logarithms. Their graphs are examined and their applications in a wide range of settings are explored. Arithmetic and geometric sequences are introduced and their applications are studied. Rates and average rates of change are introduced, and this is followed by the key concept of the derivative as an 'instantaneous rate of change'. These concepts are reinforced numerically, by calculating difference quotients both geometrically, as slopes of chords and tangents, and algebraically. Calculus is developed to study the derivatives of polynomial functions, with simple applications of the derivative to curve sketching, calculating slopes and equations of tangents, determining instantaneous velocities and solving optimisation problems.
Access to technology to support the computational aspects of these topics is assumed.

## Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques used in algebra, sequences and series, functions, graphs and calculus
- solve problems in algebra, sequences and series, functions, graphs and calculus
- apply reasoning skills in algebra, sequences and series, functions, graphs and calculus
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems.

| Unit 2 Content descriptions | | |
|---|---|---|
| **Topic 1: Exponential functions** | | |
| **Subtopics** | **Content statements** | **Textbook sections** |
| **Indices and the index laws** | • review indices (including fractional indices) and the index laws (ACMMM061) AC | 11.01 |
| | • use radicals and convert to and from fractional indices (ACMMM062) AC | 11.01, 11.02 |
| | • understand and use scientific notation and significant figures (ACMMM063) AC | 11.03 |
| **Exponential functions** | • establish and use the algebraic properties of exponential functions (ACMMM064) AC | 11.04 |
| | • recognise the qualitative features of $y = a^x$ $(a > 0)$ including asymptotes, and of its translations $y = a^x + b$ and $y = a^{x+c}$ (ACMMM065) AC | 11.05, 11.06 |
| | • identify contexts suitable for modelling by exponential functions and use them to solve practical problems (ACMMM066) AC | 11.08 |
| | • solve equations involving exponential functions using technology, and algebraically in simple cases (ACMMM067) AC | 11.07 |

| Unit 2 Content descriptions | | |
|---|---|---|
| **Topic 2: Arithmetic and geometric sequences and series** | | |
| **Subtopic** | **Statement** | **Textbook sections** |
| **Arithmetic sequences** | • recognise and use the recursive definition of an arithmetic sequence: $d = t_{n+1} - t_n$ (ACMMM068) AC | 8.01, 8.02 |
| | • use the formula $t_n = t_1 + (n-1)d$ for the general term of an arithmetic sequence and recognise its linear nature (ACMMM069) AC | 8.03 |
| | • use arithmetic sequences in contexts involving discrete linear growth or decay, such as simple interest (ACMMM070) AC | 8.03, 8.04 |
| | • establish and use the formula for the sum of the first $n$ terms of an arithmetic sequence (ACMMM071) AC | 8.04 |
| **Geometric sequences** | • recognise and use the recursive definition of a geometric sequence: $t_{n+1} = rt_n$ (ACMMM072) AC | 8.01, 8.05 |
| | • use the formula $t_n = ar^{n-1}$ for the general term of a geometric sequence and recognise its exponential nature (ACMMM073) AC | 8.06 |
| | • understand the limiting behaviour as behaviour as $n \to \infty$ of the terms $t_n$ in a geometric sequence and its dependence on the value of the common ratio $r$ (ACMMM074) AC | 8.06 |
| | • establish and use formula $S_n = \frac{t_1(r^n - 1)}{r - 1}$ for the sum of the first $n$ terms (ACMMM075) AC | 8.07 |
| | • use geometric sequences in contexts involving geometric growth or decay, such as compound interest (ACMMM076) AC | 8.07 |
| | • [Optional] establish and use the formula for the limiting sum of a geometric sequence | 8.08 |

| Unit 2 Content descriptions | | |
|---|---|---|
| **Topic 3: Introduction to differential calculus** | | |
| **Subtopic** | **Statement** | **Textbook sections** |
| **Rates of change** | • interpret the difference quotient $\frac{f(x+h) - f(x)}{h}$ as the average rate of change of a function $f$ (ACMMM077) AC | 7.01–7.04 |
| | • use the Leibniz notation $\delta x$ and $\delta y$ for the changes in the variables $x$ and $y$ (ACMMM078) AC | 7.05 |
| | • use the notation $\frac{\delta y}{\delta x}$ for the difference quotient $\frac{f(x+h) - f(x)}{h}$ where $y = f(x)$ (ACMMM079) AC | 7.05, 7.06 |
| | • interpret the ratios $\frac{f(x+h) - f(x)}{h}$ and $\frac{\delta y}{\delta x}$ as the slope or gradient of a chord or secant of the graph of $y = f(x)$ (ACMMM080) AC | 7.05, 7.06 |
| **The concept of the derivative** | • examine the behaviour of the difference quotient $\frac{f(x+h) - f(x)}{h}$ as $h \to 0$ as an informal introduction to the concept of a limit (ACMMM081) AC | 9.01, 9.02 |

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| Unit 2 Content descriptions | | |
|---|---|---|
| **Topic 3: Introduction to differential calculus** | | |
| **Subtopic** | **Statement** | **Textbook sections** |
| **The concept of the derivative** | • define the derivative as $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ (ACMMM082) AC | 9.02, 9.03 |
| | • use the Leibniz notation for the derivative: $\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$ and the correspondence $\frac{dy}{dx} = f'(x)$ where $y = f(x)$ (ACMMM083) AC | 9.03 |
| | • interpret the derivative as the instantaneous rate of change (ACMMM084) AC | 9.02, 9.05 |
| | • interpret the derivative as the slope or gradient of a tangent line of the graph of $y = f(x)$ (ACMMM085) AC | 9.02, 9.06 |
| **Computation of derivatives** | • estimate numerically the value of a derivative, for simple power functions (ACMMM086) AC | 9.02, 9.03 |
| | • examine examples of variable rates of change of non-linear functions (ACMMM087) AC | 9.05, 9.07 |
| | • establish the formula $\frac{d}{dx}x^n = nx^{n-1}$ for positive integers $n$ by expanding $(x + h)^n$ or by factorising $(x + h)^n - x^n$ (ACMMM088) AC | 9.04 |
| **Properties of derivatives** | • understand the concept of the derivative as a function (ACMMM089) AC | 10.01 |
| | • recognise and use linearity properties of the derivative (ACMMM090) AC | 10.02, 10.03 |
| | • calculate derivatives of polynomials and other linear combinations of power functions (ACMMM091) AC | 10.04 |
| **Applications of derivatives** | • find instantaneous rates of change (ACMMM092) AC | 9.05, 10.06, 12.01 |
| | • find the slope of a tangent and the equation of the tangent (ACMMM093) AC | 9.06, 10.05 |
| | • construct and interpret position–time graphs, with velocity as the slope of the tangent (ACMMM094) AC | 12.02 |
| | • sketch curves associated with simple polynomials; find stationary points, and local and global maxima and minima; and examine behaviour as $x \to \infty$ and $x \to -\infty$ (ACMMM095) AC | 12.04–12.06 |
| | • solve optimisation problems arising in a variety of contexts involving simple polynomials on finite interval domains (ACMMM096) AC | 12.07 |
| **Anti-derivatives** | • calculate anti-derivatives of polynomial functions and apply to solving simple problems involving motion in a straight line (ACMMM097) AC | 10.07, 12.03 |